Reg. No. | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

PG DEGREE END SEMESTER EXAMINATIONS - APRIL 2025.

(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: M.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
II	PART-III	CORE-4	P23MA204	ADVANCED ALGEBRA

Date & Session : 23.04.2025/AN **Time : 3 hours** **Maximum: 75 Marks**

Course Outcome	Bloom's K-level	Q. No.	SECTION – A (10 X 1 = 10 Marks) Answer ALL Questions.
CO1	K1	1.	If L is a finite extension of F and K is a subfield of L which contains F then a) $[K:F]/[L:F]$ b) $[L:F]/[K:F]$ c) there is no relation between F and L d) None of the above
CO1	K2	2.	Let F be the field, K be a finite extension of F then a) K is not a field b) K is isomorphic to F c) K is algebraic over F d) $F \cup K = K$
CO2	K1	3.	Let $f(x)$ be a polynomial over a field F, suppose $a \in F$ is a root of $f(x)$ then a) $x + a$ divides $f(x)$ b) $x - a$ divides $f(x)$ c) $x + a$ does not divides $f(x)$ d) $x - a$ does not divides $f(x)$
CO2	K2	4.	Suppose $f(x) \in F(x)$ is irreducible, if the characteristic of F is 0 then a) $f(x)$ has no roots in F b) $f(x)$ has at least one roots in F c) $f(x)$ has exactly roots in F d) $f(x)$ has no roots in any field
CO3	K1	5.	Let K be a field, F be the subfield of K then the group of automorphisms of K relative to F is a) Galois group E/F b) automorphic group of E/F c) isomorphic group of E/F d) permutation group of E/F
CO3	K2	6.	Which are the following properties being true in Galois extension a) finite and normal b) finite, normal and separable c) infinite and normal d) finite and separable
CO4	K1	7.	Any two finite fields having the same number of elements are a) Homomorphic b) Homoeomorphic c) Isomorphic d) Isometric isomorphic
CO4	K2	8.	Let F be a finite field, G be the group of non-zero elements of F then a) the additive group of F is not cyclic b) the additive group of F is cyclic c) the multiplicative group of F is not cyclic d) the multiplicative group of F is cyclic
CO5	K1	9.	The ring of integral quaternions is a) commutative but not associative b) associative but not commutative c) both commutative and associative d) neither commutative nor associative
CO5	K2	10.	Let $f(x)$ be a polynomial over a field F which of the following is necessary condition for $f(x)$ to be solvable by radicals? a) $f(x)$ must be irreducible over F b) $f(x)$ must be reducible over F c) $f(x)$ must have distinct roots d) $f(x)$ must have no repeated roots

Course Outcome	Bloom's K-level	Q. No.	SECTION – B (5 X 5 = 25 Marks) Answer ALL Questions choosing either (a) or (b)
CO1	K2	11a.	The elements $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F **(OR)**
CO1	K2	11b.	If L is the algebraic extension of K and if K is the algebraic extension of F then L is the algebraic extension of F

CO2	K2	12a.	Let $f(x) \in F(x)$ be of degree $n \geq 1$. Then there is an extension $E\ of\ F$ of degree at most $n!$ in which $f(x)$ has n roots **(OR)**
CO2	K2	12b.	For any $f(x), g(x) \in F[x\}and\ any\ \alpha \in F,$ (i) $(f(x) + g(x)' = f'(x) + g'(x)$ (ii) $\left(\alpha f(x)\right)' = \alpha f'(x)$ (iii) $\left(f(x)g(x)\right)' = f'(x)g(x) + f(x)g'(x)$
CO3	K3	13a.	If $K$is a field and if $\sigma_1, \sigma_2, \ldots . \sigma_n$ are distinct automorphisms of K then it is impossible to find elements $a_1, a_2, \ldots . a_n$ not all 0, in K such that $a_1\sigma_1(u), a_2\sigma_2(u), \ldots . a_n\sigma_n(u) = 0\ , for\ all\ u\ \in K$ **(OR)**
CO3	K3	13b.	K is a normal extension of F if and only if K is the splitting field of some polynomial over F
CO4	K3	14a.	For every prime number p and every positive integer m there exists a field having p^m elements **(OR)**
CO4	K3	14b.	If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F then we can find elements $a\ and\ b$ in F such that $1 + \alpha a^2 + \beta b^2 = 0$
CO5	K4	15a.	Suppose that the field F has all nth roots of unity and suppose that $a \neq 0\ in\ F$. Let $x^n - a \in F[x]$ and let K be its splitting filed over F. Then (i) $K = F(u)$ where u is any root of $x^n - a$ (ii) The Galois group of $x^n - a$ over F is abelian **(OR)**
CO5	K4	15b.	Let C be the field of complex numbers and suppose that the division ring D is algebraic over C the $D = C$

Course Outcome	Bloom's K-level	Q. No	**<u>SECTION – C</u> (5 X 8 = 40 Marks)** **Answer <u>ALL</u> Questions choosing either (a) or (b)**
CO1	K4	16a.	Prove that, if L is the finite extenstion of K and if K is a finite extension of F, then L is a finite extension of F Moreover, $[L: F] = [L: K][K: F]$ **(OR)**
CO1	K4	16b.	Prove that the number e is transcendental
CO2	K5	17a.	The polynomial $\mathrm{f}(x) \in F[x]$ has a multiple root if and only if $f(x)\ and\ f'(x)$ have a nontrivial common factor **(OR)**
CO2	K5	17b.	A polynomial of degree n over a field can have at most n roots in any extension field
CO3	K5	18a.	The fixed field of G is the subfield o F. And $G(K, F)$is the subgroup of the group of all automorphisms of K **(OR)**
CO3	K5	18b.	Let F_0 be the field of rational numbers. Let $\omega = e^{\frac{2\pi i}{5}}$ and ω satisfies the polynomial $x^4 + x^3 + x^2 + x + 1$ over F_0 , prove that the polynomial is irreducible over the field of rational numbers and therefore G [K,F_0] is a group of order 4
CO4	K5	19a.	Let G be a finite abelian group enjoying the property that the relation $x^n = e$ is satisfied by at most n elements of G, for every integer n. Then G is a cyclic group **(OR)**
CO4	K5	19b.	State and Prove WEDDERBURN theorem
CO5	K6	20a.	Prove that, every positive integer can be expressed as the sum of squares of four integers. **(OR)**
CO5	K6	20b.	Prove the following: 1. $x^{**} = x$ 2. $(\delta x + \gamma y)^* = \delta x^* + \gamma y^*$ 3. $(xy)^* = y^* x^*$ 4. $N(xy) = N(x)N(y)$ for all $x, y \in \mathbb{Q}\ and\ all\ real\ \delta\ and\ \gamma$